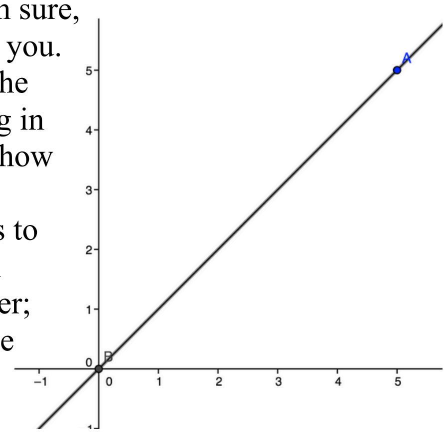
Pre-Calculus Activity Name: ______________________________

The following text is an Excerpt from Outliers: The Story of Success by Malcolm Gladwell:

A few years ago, Alan Schoenfeld, a math professor at Berkeley, made a videotape of a woman named Renee as she was trying to solve a math problem. Renee was in her mid-twenties, with long black hair and round silver glasses. In the tape, she's playing with a software program designed to teach algebra. On the screen are a *y* and *x* axis. The program asks the user to punch in a set of coordinates and then draws the line from those coordinates on the screen (through the origin). For example, when she typed in 5 on the *y* axis and 5 on the *x* axis, the computer did this (see diagram to the right). At this point, I'm sure, some vague memory of your middle-school algebra is coming back to you. But rest assured, you don't need to remember any of it to understand the significance of Renee's example. In fact, as you listen to Renee talking in the next few paragraphs, focus not on what she's saying but rather on how she's talking and why she's talking the way she is.



The point of the computer program, which Schoenfeld created, was to teach students about how to calculate the slope of a line. Slope, as I'm sure you remember (or, more accurately, as I'll bet you don't remember; I certainly didn't), is rise over run. The slope of the line in our example is 1, since the rise is 5 and the run is 5.

So there is Renee. She's sitting at the keyboard, and she's trying to figure out what numbers to enter in order to get the computer to draw a line that is absolutely vertical, that is directly superimposed over the *y* axis. Now, those of you who remember your high school math will know that this is, in fact, impossible. A vertical line has an undefined slope. Its rise is infinite: any number on the *y* axis starting at zero and going on forever. Its run on the *x* axis, meanwhile, is zero. Infinity divided by zero is not a number.

But Renee doesn't realize that what she's trying to do can't be done. She is, rather, in the grip of what Schoenfeld calls a "glorious minsconception," and the reason Schoenfeld likes to show this particular tape is that it is a perfect demonstration of how this misconception came to be resolved.

Renee was a nurse. She wasn't someone who had been particularly interested in mathematics in the past. But she had somehow gotten hold of the software and was hooked.

"Now, what I want to do is make a straight line with this formula, parallel to the *y* axis," she begins. Schoenfeld is sitting next to her. She looks over at him anxiously. "It's been five years since I did any of this."

She starts to fiddle with the program, typing in different numbers.

"Now if I change the slope that way … minus 1 … now what I mean to do is make the line go straight."

As she types in numbers, the line on the screen changes.

"Oops. That's not going to do it."

She looks puzzled.

"What are you trying to do?" Schoenfeld asks.

"What I'm trying to do is make a straight line parallel to the *y* axis. What do I need to do here? I think what I need to do is change this a little bit." She points at the place where the number for the *y* axis is. "That was something I discovered. That when you go from 1 to 2, there was a rather big change. But now if you get way up there you have to keep changing."

This is Renee's glorious misconception. She's noticed the higher she makes the *y* axis coordinate, the steeper the line gets. So she thinks the key to making a vertical line is just making the *y* axis coordinate large enough.

"I guess 12 or even 13 could do it. Maybe even as much as 15."

She frowns. She and Schoenfeld go back and forth. She asks him questions. He prods her gently in the right direction. She keeps trying and trying, one approach after another. At one point, she types in 20 (for the slope). The line gets a little bit steeper. She types in 40. The line gets steeper still.

I see that there is a relationship there. But as to why, it doesn't seem to make sense to me…. What if I do 80? If 40 gets me halfway, then 80 should get me all the way to the *y* axis. So let's just see what happens."

She types in 80. The line is steeper. But it's still not totally vertical.

"Ohhh. It's infinity, isn't it? It's never going to get there." Renee is close. But then she reverts to her original misconception.

"So what do I need? 100? Every time you double the number, you get halfway to the *y* axis. But it never gets there…" She types in 100.

"It's closer. But not quite there yet."

She starts to think out loud. It's obvious she's on the verge of figuring something out. "Well, I knew this, though … but…I knew that. For each one up, it goes that many over. I'm still somewhat confused as to why…"

She pauses, squinting at the screen.

"I'm getting confused. It's a tenth of the way to the one. But I don't want it to be…"

And then she sees it.

"Oh! It's any number up, and zero over. It's any number divided by zero!" Her face lights up. "A vertical line is anything divided by zero—and that's an undefined number. Ohhh. Okay. Now I see. The slope of a vertical line is undefined. Ahhhh. That means something now. I won't forget that!"

Over the course of his career, Schoenfeld has videotaped countless students as they worked on math problems. But the Renee tape is one of his favorites because of how beautifully it illustrates what he considers to be the secret to learning mathematics. Twenty-two minutes pass from the moment Renee begins playing with the computer program to the moment she says, "Ahhh. That means something now." That's a *long* time. "This is eighth-grade mathematics," Schoenfeld said. "If I put the average eighth grader in the same position as Renee, I'm guessing that after the first few attempts, they would have said, 'I don't get it. I need you to explain it.'" Schoenfeld once asked a group of high school students how long they would work on a homework question before they concluded it was too hard for them ever to solve. Their answers ranged from thirty seconds to five minutes, with the average answer two minutes.

But Renee persists. She experiments. She goes back over the same issues time and again. She thinks out loud. She keeps going and going. She simply won't give up. She knows on some vague level that there is something wrong with her theory about how to draw a vertical line, and she won't stop until she's absolutely sure she has it right.

Renee wasn't a math natural. Abstract concepts like "slope" and "undefined" clearly didn't come easily to her. But Schoenfeld could not have found her more impressive.

"There's a will to make sense that drives what she does," Scheonfeld says. "She wouldn't accept a superficial 'Oh yeah, you're right' and walk away. That's not who she is. And that's really unusual." He rewound the tape and pointed to a moment when Renee reacted with genuine surprise to something on the screen.

"Look" he said. "She does a double take. Many students would just let that fly by. Instead, she thought, "That doesn't jibe with whatever I'm thinking. I don't get it. That's important. I want an explanation.' And when she finally gets the explanation, she says, 'Yeah, that fits.'"

We sometimes think of being good at mathematics as an innate ability. You either have "it" or you don't. But to Schoenfeld, it's not so much ability as *attitude.* You master mathematics if you are willing to try. That's what Schoenfeld attempts to teach his students. Success is a function of persistence and doggedness and the willingness to work hard for twenty-two minutes to make sense of something that most people would give up on after thirty seconds. Put a bunch of Renees in a classroom, and give them the space and time to explore mathematics for themselves, and you could go a long way.

Discussion Questions

1. Why do you think Schoelfeld calls Renee's misconception "glorious"?

2. When Renee realizes her misconception and figures out that a slope of a vertical line is undefined she states, "That means something now. I won't forget that!" What do you think is the significance of that reaction?

3. Why didn't Schoenfeld just explain Renee's misconception to her?

4. How does this exerpt compare to the way you approach the learning of mathematics?

5. Do you agree with Schoenfeld's belief about the learning of mathematics as described below:

> "We sometimes think of being good at mathematics as an innate ability. You either have "it" or you don't. But to Schoenfeld, it's not so much ability as *attitude.* You master mathematics if you are willing to try. That's what Schoenfeld attempts to teach his students. Success is a function of persistence and doggedness and the willingness to work hard for twenty-two minutes to make sense of something that most people would give up on after thirty seconds."